Gravitation Questions

1. $F = G\frac{M_1 M_2}{r^2} = \frac{6.673\times10^{-11} \times 2.148\times10^{30} \times 4.80\times10^{27}}{(2.10\times1.496\times10^{11})^2}$

$= \underline{6.97\times10^{24}\ N}$

2. $V = \sqrt{\frac{2GM}{r}}$

$= \sqrt{\frac{2\times6.673\times10^{-11}\times3.285\times10^{23}}{2.44\times10^{6}}}$

$= 4238.8\ ms^{-1}$

$= \underline{4.24\ kms^{-1}}$

3. C

4. (a) Must supply it with at least 4.0×10^{9} J of KE.

(b) $GPE \propto \frac{1}{r}$, so if $r \Rightarrow 100r$ then $\underline{GPE \Rightarrow 10^{-2}}$.

(c) $W = F = G\frac{M_1 M_2}{r^2} = \frac{6.673\times10^{-11}\times100\times5.97\times10^{24}}{(10^{9})^2}$

$= \underline{0.040\ N}\quad (4\times10^{-2}N)$

(d) $W = F_c = m.a_c$

$a_c = \frac{F_c}{m} = \frac{0.040}{100} = \underline{4.0\times10^{-4}ms^{-2}}$

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5. $T = 24.6 \text{ hrs} = 24.6 \times 3600 = 88560 \text{ s}$

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

$$\therefore r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = \sqrt[3]{\frac{6.673\times10^{-11} \times 6.39\times10^{23} \times (88560)^2}{4\pi^2}}$$

$$= 2.04 \times 10^7 \text{ m}$$

$$= \underline{2.04 \times 10^4 \text{ km}}$$

6. $$V_N = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.673\times10^{-11} \times 1.989\times10^{30}}{(4.48\times10^{12})}}$$

$$= 5443 \text{ ms}^{-1}$$

$$= \underline{5.44 \text{ km s}^{-1}}$$

7. $$\frac{r^3}{T^2} = \frac{GM}{4\pi^2} \quad \text{and } T_{Earth} = 365.25 \times 24 \times 3600$$

$$= \underline{3.156 \times 10^7 \text{ s}}$$

$$\therefore M_S = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 \times (1.5\times10^{11})^3}{6.673\times10^{-11} \times (3.156\times10^7)^2}$$

$$= \underline{2.0 \times 10^{30} \text{ kg}}$$

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8. Acceleration due to gravity at surface of Sun:

$$g = G\frac{M_S}{r_S^2} = \frac{6.673\times10^{-11}\times 1.989\times10^{30}}{(6.96\times10^{8})^2}$$

$$= 273.99\ ms^{-2}$$

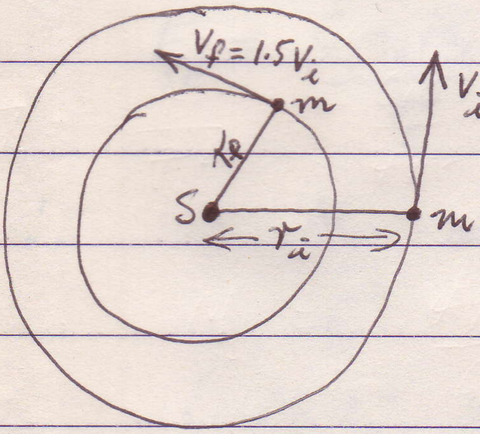
So, for new Earth: $g = G\frac{M_E}{r_E^2}$ and $\therefore r_E^2 = G\frac{M_E}{g}$

$$\therefore r_E = \sqrt{\frac{6.673\times10^{-11}\times 5.97\times10^{24}}{273.99}}$$

$$= 1.21\times10^{6}\ m$$

$\therefore$ New radius of Earth is 1.21×10^{6} m.

9. Use i for initial orbit and f for final orbit:



New orbit is smaller radius, since speed has increased.

By Kepler's 3rd Law:

$$\frac{r_i^3}{T_i^2} = \frac{r_f^3}{T_f^2}$$

$$\therefore \frac{r_f^3}{r_i^3} = \frac{T_f^2}{T_i^2} \quad \text{and} \quad T_i = \frac{2\pi}{\omega_i} = \frac{2\pi r_i}{v_i}$$

$$\text{and} \quad T_f = \frac{2\pi r_f}{v_f} = \frac{2\pi r_f}{1.5v_i} = \frac{4\pi r_f}{3v_i}$$

$$\therefore \frac{r_f^3}{r_i^3} = \left(\frac{4\pi r_f}{3v_i}\right)^2 \times \left(\frac{v_i}{2\pi r_i}\right)^2$$

$$= \frac{16\pi^2 r_f^2}{9v_i^2} \times \frac{v_i^2}{4\pi^2 r_i^2}$$

$$\therefore \frac{r_f^3}{r_i^3} = \frac{4r_f^2}{9r_i^2}$$

$$\therefore \frac{r_f}{r_i} = \frac{4}{9}$$

and so $r_f = 0.44 r_i$

So, radius of new orbit is <u>$0.44r$</u>.

10. <u>(Extension)</u> When zero net force occurs : $\underset{\sim}{F}_{SE} = \underset{\sim}{F}_{SM}$

$$F_{SE} = \frac{Gm_sM_E}{r_E^2} \quad \text{and} \quad F_{SM} = \frac{Gm_sM_M}{r_M^2} = \frac{Gm_sM_M}{(R-r_E)^2}$$

$$\therefore \frac{M_E}{r_E^2} = \frac{M_M}{(R-r_E)^2}$$

$$\therefore M_M r_E^2 = M_E(R-r_E)^2$$

$$\therefore M_M r_E^2 = M_E(R^2 - 2Rr_E + r_E^2)$$

$$\therefore M_M r_E^2 = M_E r_E^2 - 2M_E R r_E + M_E R^2$$

$$\therefore (M_E - M_M)r_E^2 - 2M_E R r_E + M_E R^2 = 0$$

$$\therefore (5.97\times10^{24} - 7.35\times10^{22})r_E^2 - (2\times5.97\times10^{24}\times3.844\times10^8)r_E$$
$$+ (5.97\times10^{24}\times3.844^2\times10^{16}) = 0$$

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$\therefore \; 5.8965 \times 10^{24} r_E^{\,2} - 4.5897 \times 10^{33} r_E + 8.8215 \times 10^{41} = 0$

Now we use the quadratic formula to solve for r_E:

$$\therefore \; r_E = \frac{4.5897 \times 10^{33} \pm \sqrt{(4.5897 \times 10^{33})^2 - (4 \times 5.8965 \times 10^{24} \times 8.8215 \times 10^{41})}}{2 \times 5.8965 \times 10^{24}}$$

$$= \frac{4.5897 \times 10^{33} \pm 5.0888 \times 10^{32}}{1.1793 \times 10^{25}}$$

$$= 4.323 \times 10^{8} \text{ m and } \underline{3.460 \times 10^{8} \text{ m}}$$

$\therefore$ Distance from Earth where spacecraft will experience zero net force is 3.460×10^{8} m, since 4.323×10^{8} m > Earth-Moon distance and $\therefore$ cannot be an answer to this question.

PS - You might like to consider what the other solution means here. It does have a physical meaning. Consider the 3-dimensional nature of the gravity fields of Earth + Moon.